2 Algebra

In this section you will be reminded about the different topics you need to be familiar with to be successful in a module on business analytics. Many, if not all of these topics you will have studied previously, but you may need a little refresher. It is important that you don't skip this section as the concepts we will be looking at are fundamental and will help you immensely in the later sections.

2.1 Substitution

We use formulae all of the time in business analytics. For example we can use a formula to calculate the supply and demand of an item. Consider the following supply and demand equations.

$$Q_d = 30 - 3P$$
$$Q_s = -40 + 2P$$

where Q_d is the demand of the item and Q_s is the supply of the item. You can see that they are both dependent on another variable P which in this example is the price of the item. We can see that if we change the value of P both the values of Q_d and Q_s will change as a consequence. We therefore say that P is the independent variable and both Q_s and Q_d are the dependent variables as they change as a consequence of P changing.

For example, I may want to know the demand on an item at a certain price. I might ask myself "what is the demand going to be if I price the item at £2.50?" To find the demand I would replace the P in our demand equation with the value 2.50 and the resulting answer would give the demand. The demand would therefore be:

$$Q_d = 30 - 3 \times 2.50 = 22.50$$

Generally when we are performing substitution we replace a letter with a specific value.

Example 2.1.1

You are given the expression
$$y = x^2 + 3x - 2$$
Calculate the value of y for the following values of x.
1. $x = 2$
2. $x = 0$
3. $x = -3$

Solution

1. When $x = 2$ we swap any 'x's in our expression with the value 2, i.e.

$$y = x^2 + 3x - 2 = 2^2 + 3 \times 2 - 2 = 4 + 6 - 2 = 8.$$

When $x = 2$, $y = 8$.

2. When $x = 0$ we are again required to swap any 'x's in our expression with the value 0.

$$y = x^2 + 3x - 2 = 0^2 + 3 \times 0 - 2 = 0 + 0 - 2 = -2.$$

When $x = 0$, $y = -2$.

3. When $x = -3$ we are still required to swap any 'x's in our expression with the value -3, but we should take extra care. When subsituting a negative value into an expression you should write the value inside brackets. Whilst this may not seem to be significant, you will find that if using your calculator and you forget to put your brackets in, your calculator will give you the wrong answer. If we use the brackets carefully we will find

$$y = x^2 + 3x - 2 = (-3)^2 + 3(-3) - 2 = 9 - 9 - 2 = -2.$$

When $x = -3$, $y = -2$.

Example 2.1.2

You are given the equation:

$$s = (12 - 4.9t)t$$

Calculate the value of s for the following values of t.

1. $t = 4$
2. $t = -2$
3. $t = -6$

Solution

1. When $t = 4$ we swap any 't's in our expression with the value 4.

$$s = (12 - 4.9t)t = (12 - 4.9 \times 4) \times 4 = -30.4$$

When $t = 4$, $s = -30.4$.

2. When $t = -2$ we are again required to swap any 't's in our expression with the value -2 and recall that when we substitute in a negative value, we should write it within brackets.

$$s = (12 - 4.9t)t = (12 - 4.9 \times (-2)) \times (-2) = -43.6$$

When $t = -2$, $s = -43.6$.

3. When $t = -6$ we swap any 't's in our expression with the value -6 to find

$$s = (12 - 4.9t)t = (12 - 4.9 \times (-6)) \times (-6) = -248.4.$$

When $t = -6$, $s = -248.4$.

2.1.1   Expressions

Sometimes we may be required to derive our own expression and then subsitutute a value into it. Consider the following example.

Example 2.1.3

A taxi firm charges £0.80 per mile in addition to a fixed charge of £4.20. Write down a formula for the cost C of hiring a taxi to travel m miles and calculate how much it would cost for a 12 mile journey.

If we travel just one mile in the taxi it would cost us $C = 4.20 + 0.80 = £5$. If we were to travel two miles in the taxi it would cost us $C = 4.20 + 2 \times 0.80 = £5.80$. If we were to travel three miles it would cost a total of $C = 4.20 + 3 \times 0.80 = £6.60$. The rule to find the cost of the taxi ride is £4.20 + 0.8 multiplied by the number of miles we have travelled, leading to the following for the cost of the taxi:

$$C = 4.20 + m \times 0.80$$

To answer the question with regards to the cost of the taxi ride for a 12 mile journey we subsitute $m = 12$ into the expression to find:

$$C = 4.20 + 12 \times 0.80 = 4.20 + 9.60 = £13.80.$$

2

Confidence Builder Questions: Set 2.1

1. Given that $a = 3$ and $b = 2$, calculate the value of $c = a^2 - 2b$.

2. Given that $x = -3$ and $y = 2$ calculate the value of $z = x^2 + y^2$.

3. Given that $f = -5$ and $g = -2$ calculate the value of $h = -2f - 3g$.

4. Given that $x = 4$ and $y = -3$ calculate the value of $z = (x^2 - y^2)/(x - y)$

5. The demand function of a certain product us given by $Q_d = 40 - 2P$. What would the demand be equal to if the price $P = 2.50$?

6. You are charged 50p per unit of electricity used in your factory with a fixed charge of £22. Write down a formula for the cost C of using n units of electricty and calculate how much it would cost to use 1200 units of electricity.

7. Work out the value of $p(q - 3)/4$ when $p = 2$ and $q = -7$.

8. Given that $A = h(x+10)/2$, with $A = 27$ and $h = 4$, calculate the value of x.

9. Given that $h = 5t^2 + 2$, calculate the value of h when $t = -2$.

10. Given that $h = 5t^2+2$, calculate the value of t when $h = 47$.

2.2   Algebraic Equations

An algebraic equation is an equation involving powers of an unknown variable, typically represented by x but any other letter can be used with t, p and q commonly used in business analytics. For example, the following are classed as algebraic expressions:

$$x^5 + 4x^2 - 3x + 2 = 0, \quad q = p^2 - p + 2, \quad s = t^3 - 2t + 4 \quad y + 3 = 12$$